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UBC Number Theory Seminar: October 13, 2021

Speaker: Anwesh Ray (UBC Vancouver)
Title: Arithmetic statistics and diophantine stability for elliptic curves
Abstract: In 2017, B. Mazur and K. Rubin introduced the notion of diophantine stability for a variety defined over a number field. Given an elliptic curve E defined over the rationals and a prime number p, E is said to be diophantine stable at p if there are abundantly many p-cyclic extensions $L/\mathbb{Q}$ such that $E(L) = E(\mathbb{Q})$. In particular, this means that given any integer $n > 0$, there are infinitely many cyclic extensions with Galois group $\mathbb{Z}/p^n\mathbb{Z}$, such that $E(L) = E(\mathbb{Q})$. It follows from more general results of Mazur-Rubin that E is diophantine stable at a positive density set of primes p.

In this talk, I will discuss diophantine stability of average for pairs (E, p), where E is a non-CM elliptic curve and $p \geq 11$ is a prime number at which E has good ordinary reduction. First, I will fix the elliptic curve and vary the prime. In this context, diophantine stability is a consequence of certain properties of Selmer groups studied in Iwasawa theory. Statistics for Iwasawa invariants were studied recently (in joint work with collaborators). As an application, one shows that if the Mordell Weil rank of E is zero, then, E is diophantine stable at 100% of primes p. One also shows that standard conjectures (like rank distribution) imply that for any prime $p \geq 11$, a positive density set of elliptic curves (ordered by height) is diophantine stable at p. I will also talk about related results for stability and growth of the p-primary part of the Tate-Shafarevich group in cyclic p-extensions.