UBC Number Theory Seminar: November 17, 2021

Speakers: Ethan White and Chi Hoi Yip (UBC)

Title: The number of directions determined by a Cartesian product in the affine Galois plane

Abstract: The directions determined by a subset $U \subset AG(2, p)$ is the set of slopes formed by pairs of points from U. For $U = A \times B$, a Cartesian product, we give a new lower bound on the number of directions determined by U. Combining this result with estimates on exponential sums, we make progress on the Paley graph conjecture (a double character sum estimate).

When $A = B$ is an arithmetic progression, we give an asymptotic formula for the number of directions. Our method involves computing an asymptotic formula for the number of solutions to the Diophantine equation $ad + bc = p$.

Joint work with Daniel Di Benedetto, Greg Martin, and József Solymosi.